

#8: Let $T: \mathcal{P}(\mathbb{R}) \rightarrow \mathcal{P}(\mathbb{R})$ be a map and $F = \mathbb{R}$. Then which of the foll. is/are linear transformation?

(A) $T(z) = \bar{z}$ ✓

$$T(\alpha z_1 + z_2) = \overline{\alpha z_1 + z_2} \\ = \alpha \bar{z}_1 + \bar{z}_2$$

$$\alpha T(z_1) + T(z_2) = \alpha \bar{z}_1 + \bar{z}_2$$

(C) $T(z) = z + i$ ✗

$$T(\alpha z_1 + z_2) = \alpha z_1 + z_2 + i$$

$$\alpha T(z_1) + T(z_2) = \alpha(z_1 + i) + z_2 + i = \alpha z_1 + z_2 + \alpha i + i$$

(B) $T(z) = z$ ✓

(D) $T(z) = 0$ ✓

#9: Let $T: \mathcal{P}_n(\mathbb{R}) \rightarrow \mathcal{P}_n(\mathbb{R})$ be a map and $F = \mathbb{R}$. Then which of the foll. is/are l.t.

(A) $T(P(x)) = P'(x)$ ✓

$$T(\alpha P(x) + Q(x)) = \alpha P'(x) + Q'(x) = \alpha T(P(x)) + T(Q(x))$$

(B) $T(P(x)) = P'(0)$ ✓

(C) $T(P(x)) = \int_0^x P(x) dx$

$$T(\alpha P(x) + Q(x)) = \int_0^x (\alpha P + Q) dx = \int_0^x \alpha P dx + \int_0^x Q dx$$

$$\therefore T(x^n) = \int_0^x x^n dx = \frac{x^{n+1}}{n+1} \Big|_0^x = \frac{x^{n+1}}{n+1} \notin \mathcal{P}_n(\mathbb{R})$$